MECHANICS

When treated as a singular noun, The branch of applied mathematics dealing with motion and forces producing motion. The machinery or working parts of something.

ISAAC NEWTON (1643-1727)

THE WORLD ISAAC NEWTON WAS BORN INTO

- The left-over thought of all matter's being composed of mixtures of air, earth, fire and water was expanded by adding heat, life-force and electricity as matter components.
- ALGEBRA was a young intellectual activity. Geometry was the dominant branch of mathematics. JOHANNES KEPLER (1571-1630) showed that the planetary orbits were ELLIPSES rather than circles, with the sun at one of the foci. A line joining the sun to each planet sweeps out equal areas of the ellipse during equal periods of time. The square of each planet's period is proportional to the cube of its major axis.

PHILOSOPHIÆ NATURALIS PRINCIPIA MATHEMATICA.

Autore *JS. NEWTON*, *Trin. Coll. Cantab. Soc.* Matheseos Professore *Lucasiano*, & Societatis Regalis Sodali.

IMPRIMATUR.
S. PEPYS, *Reg. Soc.* PRÆSES.
Julii 5. 1686.

LONDINI,

Jussu *Societatis Regiæ* ac Typis *Josephi Streater*. Prostat apud plures Bibliopolas. *Anno* MDCLXXXVII.

ISAAC NEWTON'S "PRINCIPIA" (1686)

- His three basic premises of MECHANICS are (1) All bodies remain at rest or in uniform motion unless acted upon by a force. (2) Force was equal to mass time acceleration, F=ma. (3) For every action there is an equal and opposite reaction.
- To exercise these premises he provided the tool of ORDINARY DIFFERENTIAL CALCULUS.
- He proposed the LAW OF GRAVITATION, namely that the attractive force between a MASS m_1 and another MASS m_2 was equal to a constant times the product of those masses divided by the square of the distance separating the two masses, $F_g=Gm_1m_2/r^2$. (G is equal to 6.6732 X 10^{-11} newton-meter^2/Kg^2)
- NEWTON believed in ABSOLUTE SPACE-TIME.

CONSEQUENCES OF "PRINCIPIA"

- All of Kepler's Laws could be derived from Newton's premises.
- Velocity became the first derivative of position and acceleration the second derivative. Mechanics became an exact science.
- Using the Principia premises, gravitational law and calculus, if some intellect could learn the masses, positions and velocities in absolute space-time of all the bodies in the universe at the same instant, it would be theoretically possible to calculate the entire past history and future of those bodies, thereby denying the existence of FREE WILL.
- Newton was a strict MATTERIALIST. He did not believe in the HOLY GHOST, and had to receive a ROYAL EXEMPTION to join the Cambridge faculty without taking the Nicean Oath.

ELABORATION OF NEWTON’S DISCOVERIES

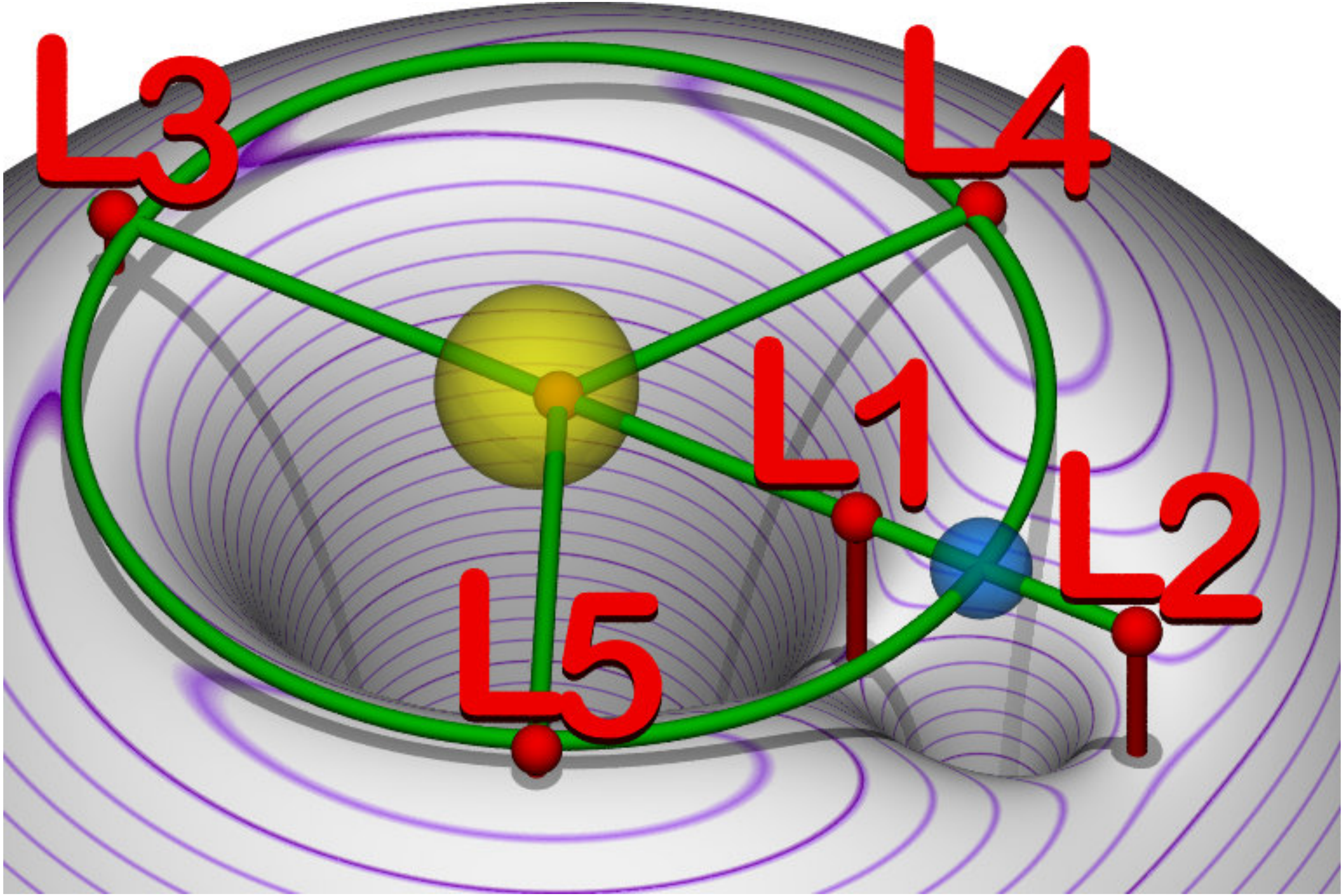
- If it was not possible to know the masses, positions and velocities of all bodies at the same time, perhaps something could be learned with lesser information dealing with a smaller portion of the universe.
- LAGRANGE’s formulation of mechanics
- HAMILTON’s formulation of mechanics.
- Principle of LEAST ACTION.
- THERMODYNAMICS.
- ACTION-AT-A-DISTANCE considerations.

GIUSEPPE-LUIGI LAGRANGIA (1736-1813)

JOSEPH LOUIS LAGRANGE.

JOSEPH-LOUIS LAGRANGE

- Although born and educated in Turin, his great grandfather was a French military officer who married a local woman. As a mathematician he was virtually self-taught. At 19 he was appointed as assistant professor at Turn's military academy, becoming the first person to teach calculus at an engineering college. At 30 (1766) he was appointed to the Prussian Academy in Berlin to succeed LEONHARD EULER (1707-1783). Here he published *Mecanigue analutique* in 1788. At 51 (1781) he was appointed to the French Academy of Sciences in Paris and in 1794 became the first faculty of the newly formed Ecole Polytechnique, and was appointed senator in 1799.


L3
L4
L1
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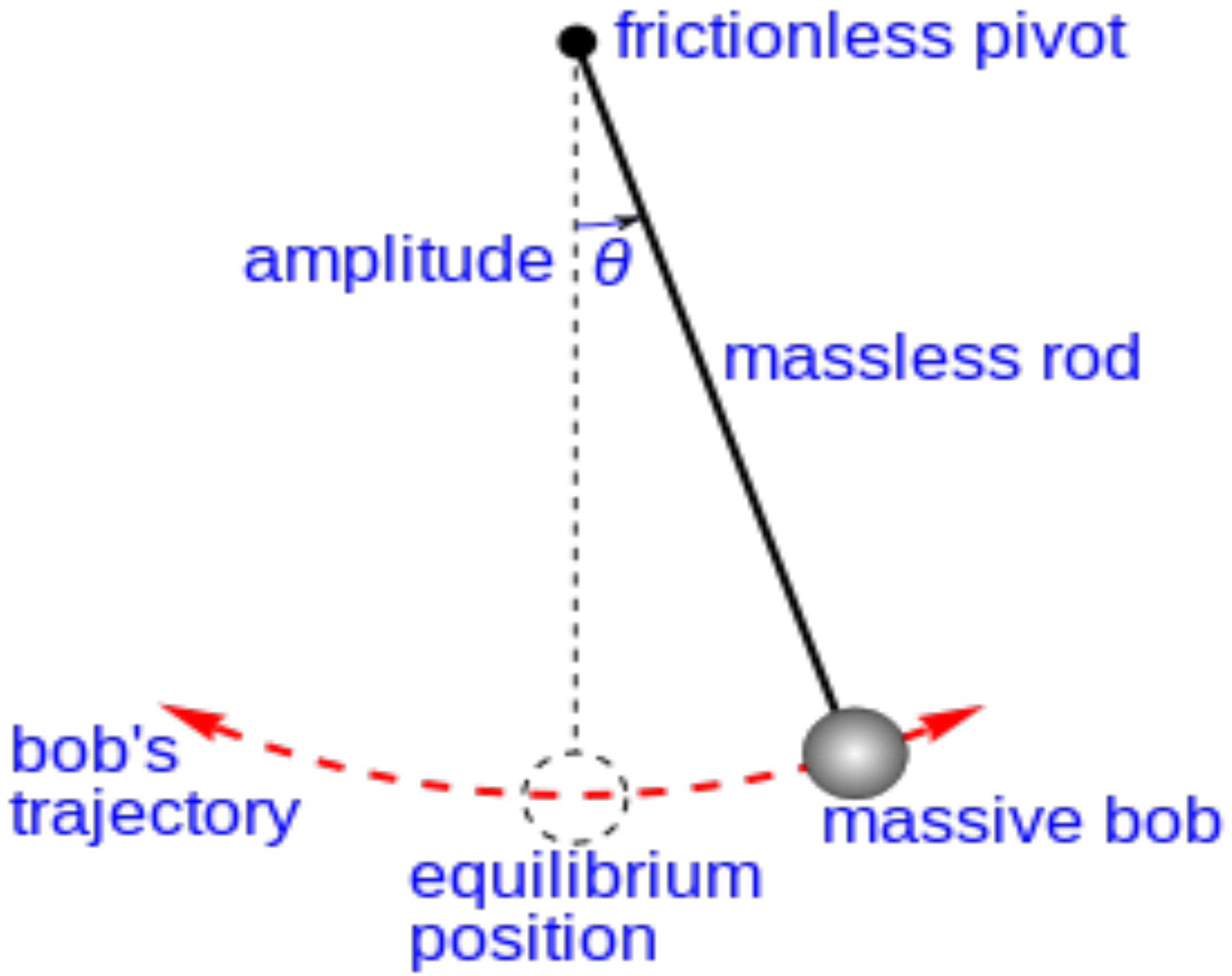
L5

LAGRANGIAN MECHANICS

- In Lagrangian mechanics, the trajectory of a system of particles in space and time is derived by solving the Lagrange equations in one of two forms: either the *Lagrange equations of the first kind*, which treat constraints explicitly as extra equations, or the *Lagrange equations of the second kind*, which incorporate the constraints directly by judicious choice of generalized coordinates. In each case, a mathematical function called the **Lagrangian** is a function of the generalized coordinates, their time derivatives, and time, and contains all the information about the dynamics of the system needed to solve its trajectory through space-time.

THE LAGRANGIAN

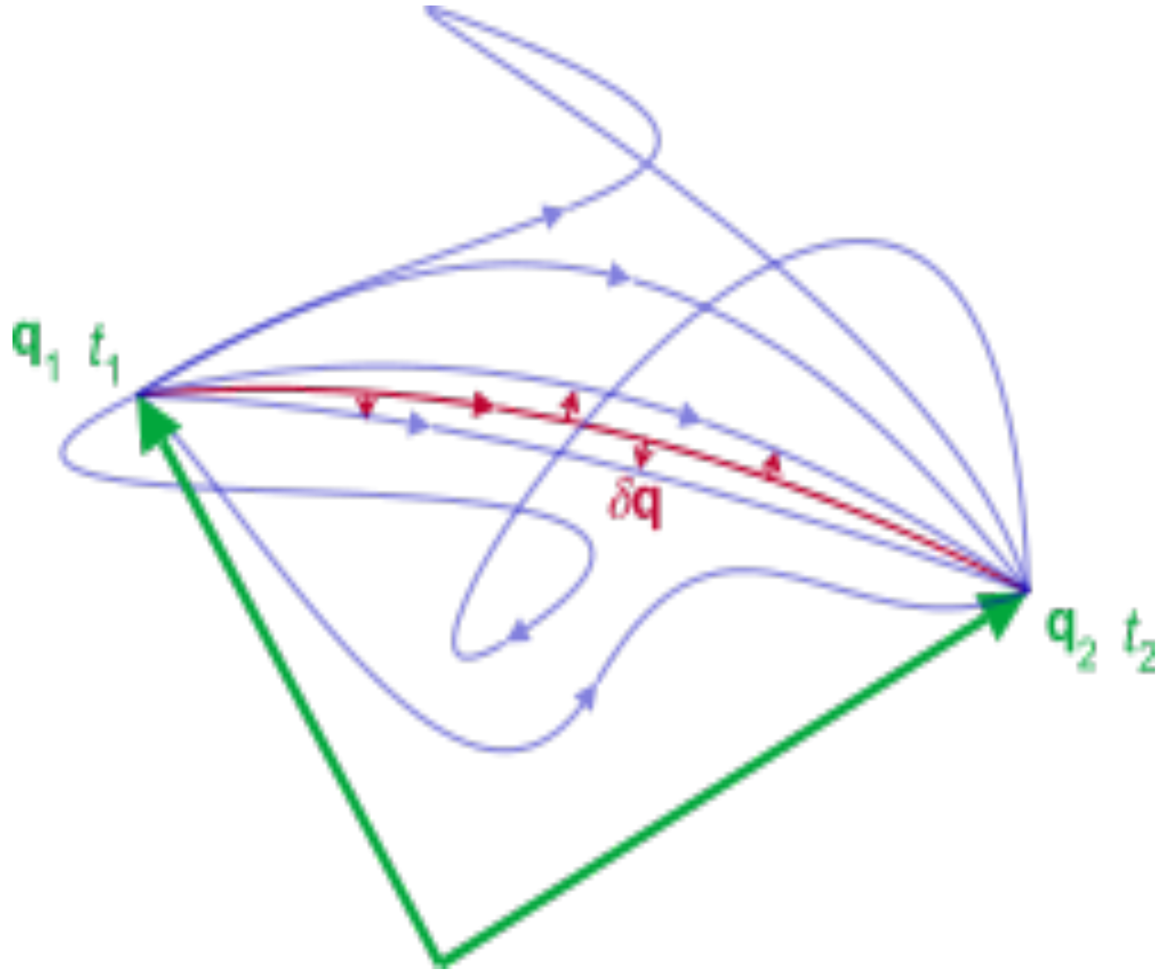
- L = T – V   The LAGRANGIAN (L) is the difference between the kinetic energy of a system of particles (T) and the potential energy of the entire system (V).
- There are two kinds of applications for the Lagrangian, the first treats constraints explicitly, the second (the kind that keeps students up at night) uses ***generalized coordinates*** such that the number of coordinates exactly equals the number of degrees of freedom.


frictionless pivot
amplitude
θ
massless rod
bob's
trajectory
equilibrium
position
massive bob

WILLIAM ROWAN HAMILTON (1805-1865)

THE HAMILTONIAN

- Hamilton built his system on the total system energy H (the Hamiltonian) = T (kinetic energy) + V (potential energy). Keeping all constraints explicit.
- PHASE SPACE (also called CONFIGURATION SPACE) is a space that has a dimension for each dimension of each of the particles in the system being studied. Each point in this multidimensional space represents a possible condition of the system. For each point there is a value of the LAGRANGIAN. As the system evolves the point describing it moves through this space at a particular velocity.
- HAMILTON'S PRINCIPLE is the statement that the TIME INTEGRAL OVER THE PASSAGE OF THE EVOLVING SYSTEM THROUGH VARIOUS VALUES OF THE LAGRANGIAN IS STATIONARY FOR THE PATH TAKEN WITH RESPECT TO ADJOINING PATHS. Also called the PRINCIPLE OF LEAST ACTION.

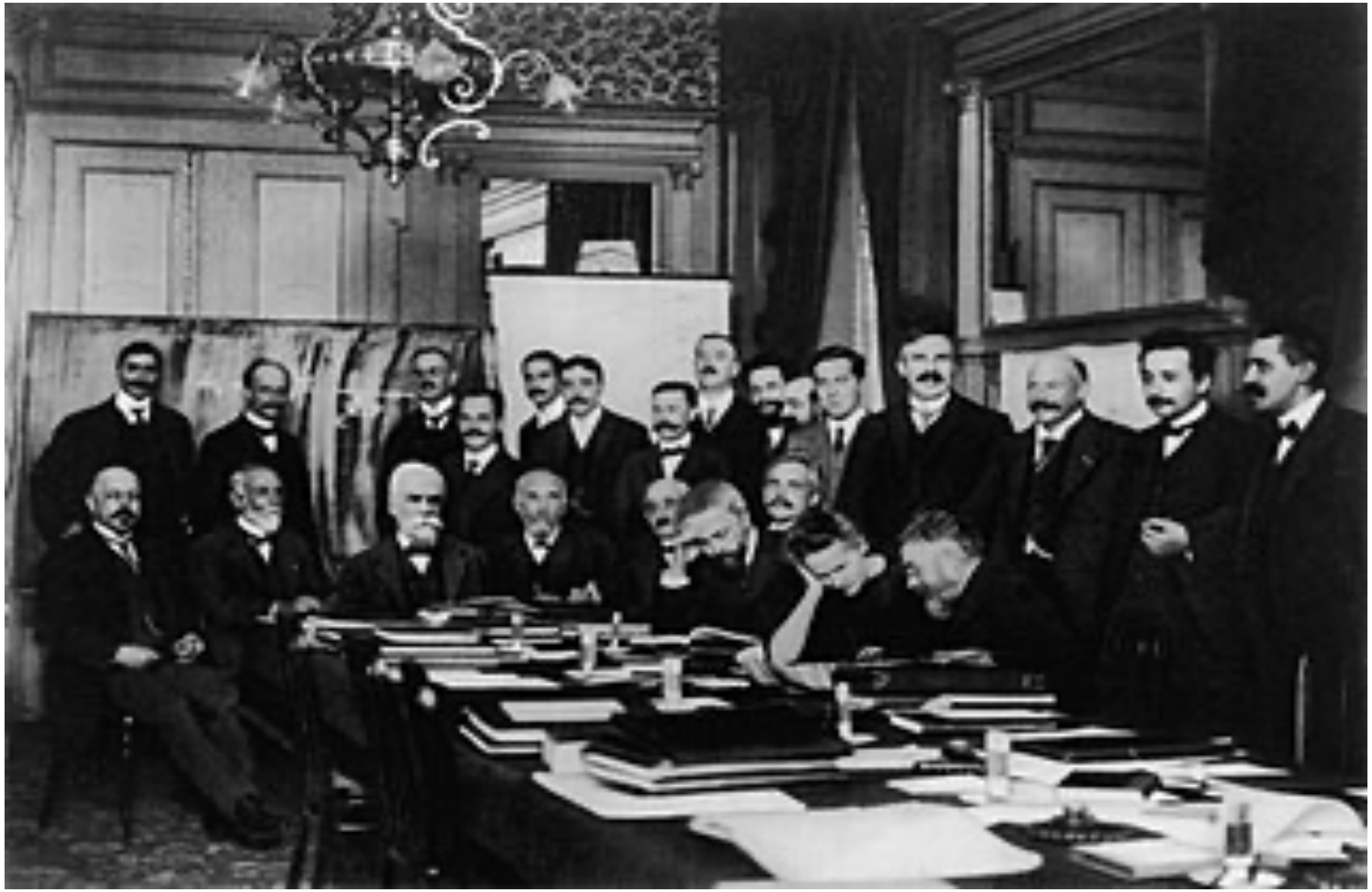


$\mathbf{q}_1$ t_1
$\delta\mathbf{q}$
$\mathbf{q}_2$ t_2

PIERRE LOUIS MAUPERTIUS (1698-1759)

- "The laws of movement and of rest deduced from this principle being precisely the same as those observed in nature, we can admire the application of it to all phenomena. The movement of animals, the vegetative growth of plants ... are only its consequences; and the spectacle of the universe becomes so much the grander, so much more beautiful, the worthier of its Author, when one knows that a small number of laws, most wisely established, suffice for all movements."— *Pierre Louis Maupertuis*
- *Given that any particle begins at position x_1 at time t_1 and ends at position x_2 at time t_2, the physical trajectory that connects these two endpoints is an extremum of the action integral.*

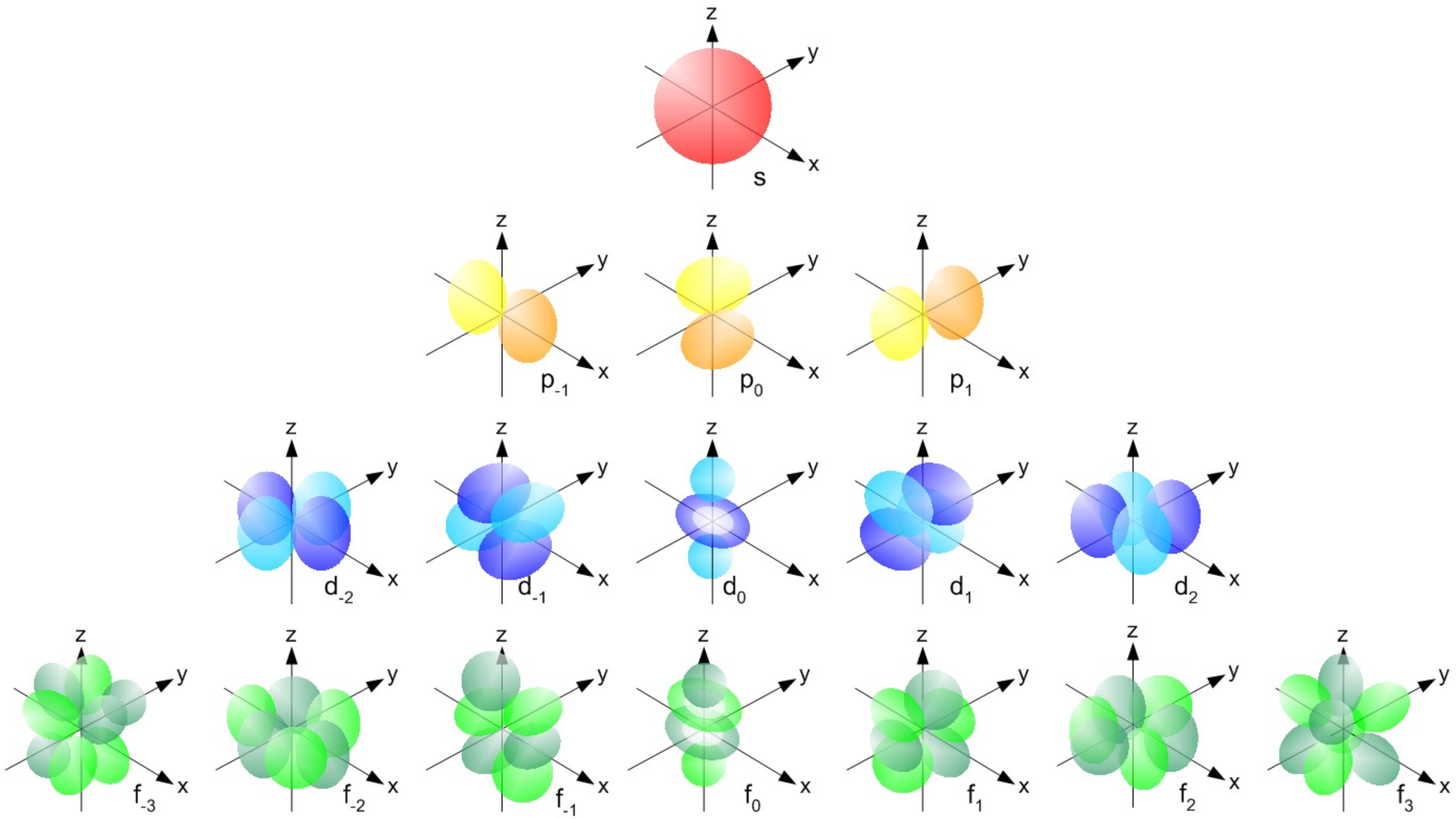
SOLWAY CONFERENCES 1911-1928



FORD’S FIRST “STARTLING IDEA”

- “…the realization that the fundamental laws of nature are laws of probability, not laws of certainty…”
- No more F=ma, no more ”the particle is at point x,y,z”
- ψ (pronounced PSI) is the Greek letter chosen to indicate a measure of the probability a particle is at a location x,y,z at time t. $\psi(x, y, z, t)$ is a FUNCTION of space and time coordinates.

WAVE FUNCTIONS



QUANTIZATION, THE SECOND IDEA

- A particle cannot have just any value of its motion, and the accuracy with which it can be measured is limited by Planck's constant.
- The key formulae of the new physics will not give ranges of values but will yield only a list of possible values. Such mathematics was already known: EIGEN VALUE equations. These equations were of the form of an OPERATOR working on a wave function the result being equal to the same function times a constant. "H" is the HAMILTONIAN OPERATOR.
- $H\psi(x, y, z, t) = (constant)\psi\ (x, y, z, t)$
- If you can be clever enough to figure out what the operators are you can calculate the wavefunctions and constants.

WHAT PARAMETERS HAVE TO BE MET?

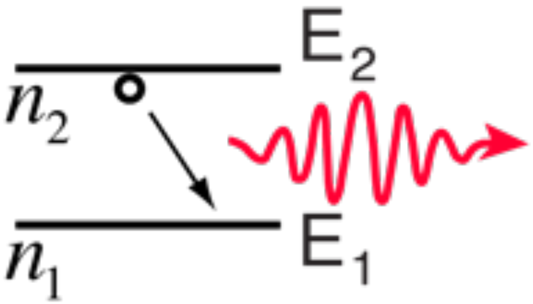
- The PERIODIC TABLE
- The EMISSION LINE WAVELENGTHS measured during the 19th century from excited atoms of all the known chemical elements.

Periodic Table of the Elements

ORBITAL SPIN OF ELECTRONS

- Eigen functions have a parameter, l, which is associated with the angular momentum of the the electron about the nucleus. They have another parameter, n, that is called the PRINCIPAL quantum number. The correlation between the intrinsic spin of the electron and l is such that for each value of l there 2(2l+1) places for the electron.
- So, l=0 gives 2 places, l=1 gives 6 places, l=2 gives 10 places and l=3 gives 14 places. Count the number of columns in each row.

The Bohr model for an electron transition in hydrogen between quantized energy levels with different quantum numbers n yields a photon by emission with quantum energy:



A downward transition involves emission of a photon of energy:

$$E_{photon} = h\upsilon = E_2 - E_1$$

Given the expression for the energies of the hydrogenic electron states for atoms of atomic number Z:

$$h\upsilon = \frac{Z^2 m e^4}{8h^2\varepsilon_0^2}\left[\frac{1}{n_1^2} - \frac{1}{n_2^2}\right] = -13.6Z^2\left[\frac{1}{n_1^2} - \frac{1}{n_2^2}\right]eV$$

PIERRE-SIMON LAPLACE (1749-1827)

THE LAPLACIAN OPERATOR

$$\triangle = \nabla \cdot \nabla = \nabla^2 = \left[\frac{\partial}{\partial x_1}, \cdots, \frac{\partial}{\partial x_N}\right] \begin{bmatrix} \frac{\partial}{\partial x_1} \\ \vdots \\ \frac{\partial}{\partial x_N} \end{bmatrix} = \sum_{n=1}^{N} \frac{\partial^2}{\partial x_n^2}$$

EIGEN OPERATOR FOR KINETIC ENERGY

- Minus hbar times the LAPLACIAN divided by 2 times the particle mass.

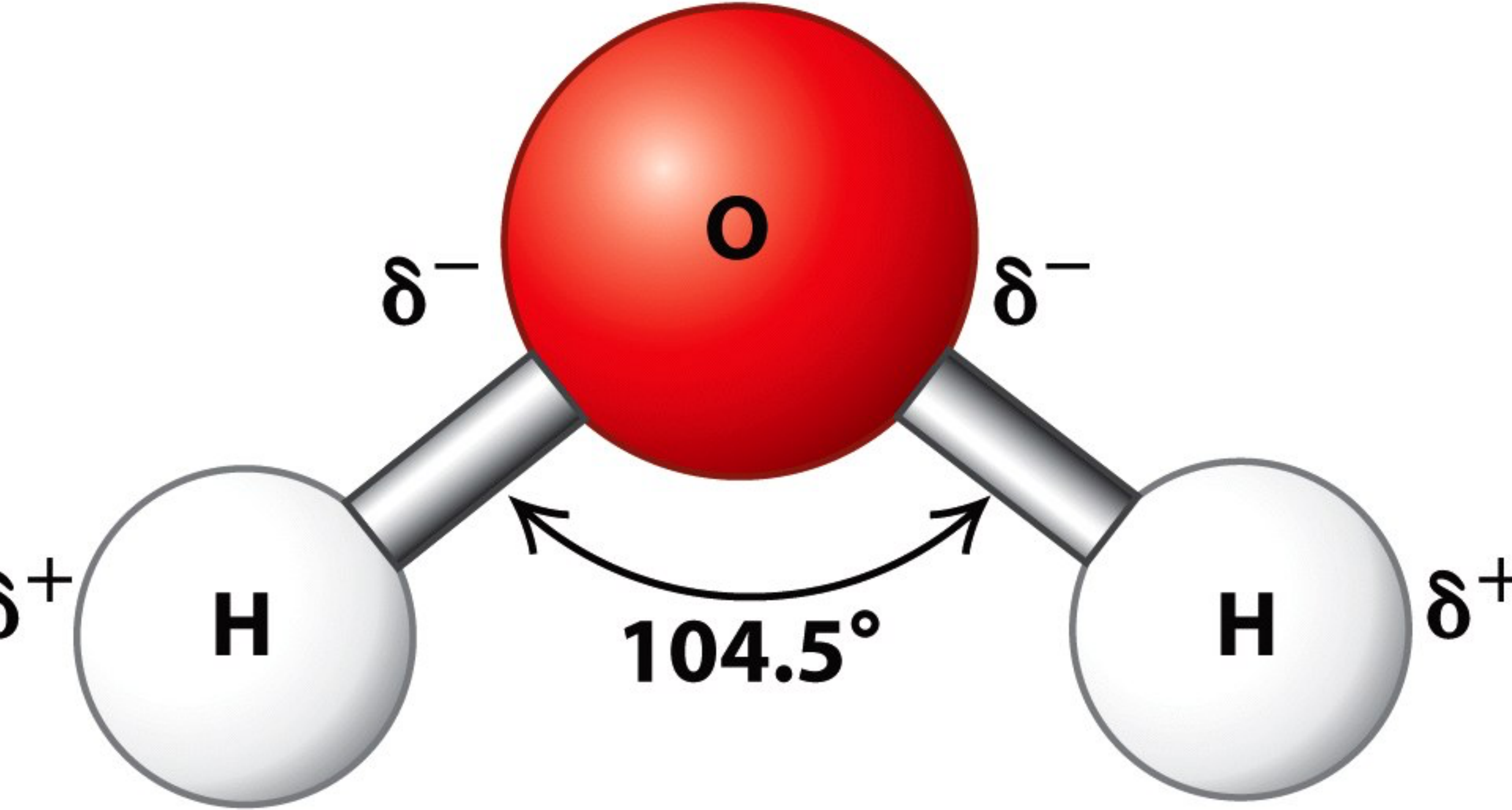
EIGEN OPERATOR FOR POTENTIAL ENERGY

- No calculus is necessary. Just use the coulomb energy of an electron in the vicinity of a nucleus.
- Ze^2/r is the Eigen operator for potential energy of an electron inside an atom.

ATOMIC EIGEN VALUE EQUATIONS

- A constant times the Laplacian operator operating on the wave function plus the coulomb energy times the wave function equals the EIGEN VALUE times the wave function. Since this is the operator for energy, the eigen values are the allowed energies of an electron inside the atom.
- $H\psi = (constant/n^2)\psi$ VOILA! Balmer's constants.

LINUS PAULING (1901-1994)


O
δ−
δ−
δ+
H
H
δ+
104.5°

An atom undressed

Electrons have quantum properties so can be in many places at once, making it hard to image atoms. By combining snapshots of many electrons at once, hydrogen atoms have now been imaged at four energy levels (increasing from a to d)